Section 3.1

Definition of Extrema: Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f on I** when $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of f on I** when $f(c) \geq f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values,** or **extrema**, of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum,** or the **global minimum** and **global maximum**, on the interval. Extrema can occur at interior points or endpoints of an interval. Extrema that occur at the endpoints are called **endpoint extrema.**

Definition of Relative Extrema:

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f, or you can say that f has a **relative maximum at $\big(c, f(c)\big)$.**
2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f, or you can say that f has a **relative minimum at $\big(c, f(c)\big)$.**

Definition of Critical Number: Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.

Relative Extrema Only Occur at Critical Numbers: If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f.

Guidelines for Finding Extrema on a Closed Interval: To find the extrema of a continuous function f on a closed interval $[a, b]$, use these steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

1) Let $f(x) = \frac{4(x-1)}{x^2}$. Use your graphing calculator to find the relative maximum of $f(x)$ on the interval $[0, 5]$.

a) Find $f'(x)$ at the relative maximum.

2) Find all of the relative extrema for the function $g(x) = \cos x$ on the interval $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$.

3) Find the extrema of $f(x) = 3x^3 + 3x^2 - 3x + 1$ on the interval $[-2, 1]$. It may help to graph the function on your calculator with the window set to the given interval.

4) Find the extrema of $g(x) = -2x - 5x^{2/5}$ on the interval $[-2, 1]$.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #3, 9, 14, 25, 34